Scope of the Entrance Exam

Mathematics

SWPS University

July 9, 2025

Below, you will find the expected learning outcomes, which will be covered during the math entrance exam to SWPS University.

Detailed Curriculum Content

I. Real Numbers

The student:

1. performs operations (addition, subtraction, multiplication, division, exponentiation, root extraction, logarithms) within the set of real numbers;

2. conducts simple proofs related to divisibility of integers and remainders from division, such as:

   (a) proof of divisibility by 24 of the product of four consecutive natural numbers,

   (b) proof of the property: if a number when divided by 5 leaves a remainder of 3, then its cube leaves a remainder of 2 when divided by 5;

3. applies properties of roots of any degree, including odd-degree roots of negative numbers;

4. applies the relation between roots and powers and rules for operations with exponents and roots;

5. applies monotonicity properties of exponentiation, in particular: if $x < y$ and $a > 1$, then $a^x < a^y$, and if $x < y$ and $0 < a < 1$, then $a^x > a^y$;

6. uses the concept of an interval on the number line and marks intervals on it;

7. applies the geometric and algebraic interpretation of absolute value, solves equations and inequalities such as:

$$|x + 4| = 5, \quad |x - 2| < 3, \quad |x + 3| \geq 4;$$

8. uses properties of exponentiation and root extraction in practical contexts, including compound interest, savings, and loan cost calculations;

9. applies the relation between logarithms and exponents, uses formulas for logarithms of products, quotients, and powers.

II. Algebraic Expressions

The student:

1. uses Pascal's triangle and applies algebraic identities such as:

$$(a+b)^2, \quad (a-b)^2, \quad a^2-b^2, \quad (a+b)^3, \quad (a-b)^3, \quad a^3-b^3, \quad a^n-b^n;$$

2. performs addition, subtraction, and multiplication of polynomials in one or more variables;

3. factors out monomials from algebraic sums;

4. factors polynomials using common factor extraction and grouping terms, in cases not more difficult than:

$$\text{factoring the polynomial } W(x) = 2x^3 - \sqrt{3}x^2 + 4x - 2\sqrt{3};$$

5. finds integer roots of polynomials with integer coefficients;

6. performs division of a polynomial in one variable $W(x)$ by a binomial of the form $x - a$;

7. performs multiplication and division of rational expressions;

8. performs addition and subtraction of rational expressions in cases not more difficult than:

$$\frac{1}{x+1} - \frac{1}{x}, \quad \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}, \quad \frac{x+1}{x+2} + \frac{x-1}{x+1}.$$

III. Equations and Inequalities

The student:

1. transforms equations and inequalities in an equivalent way;

2. interprets contradictory and identity equations and inequalities;

3. solves linear inequalities with one variable;

4. solves quadratic equations and inequalities;

5. solves polynomial equations reducible to quadratic equations, in particular biquadratic equations;

6. solves polynomial equations of the form $W(x) = 0$ for polynomials reducible to factored form or transformable to factored form by extracting common factors or grouping terms;

7. solves rational equations of the form $\dfrac{V(x)}{W(x)} = 0$, where $V(x)$ and $W(x)$ are written in factored form.

IV. Systems of Equations

The student:

1. solves systems of linear equations with two unknowns, provides a geometric interpretation of consistent, inconsistent, and dependent systems;

2. applies systems of equations to solve word problems;

3. solves systems using substitution where one equation is linear and the other is quadratic, in the form:

$$\begin{cases} ax + by = e \\ x^2 + y^2 + cx + dy = f \end{cases} \quad \text{or} \quad \begin{cases} ax + by = e \\ y = cx^2 + dx + f \end{cases}$$

V. Functions

The student:

1. defines a function as a mapping where each element from the domain is assigned to exactly one element in the codomain, using verbal description, table, graph, or formula (also using different formulas on different intervals);

2. calculates the value of a function given by an algebraic formula;

3. reads and interprets values of functions defined by tables, graphs, formulas, etc., also in situations involving repeated use of the same or multiple sources of information;

4. determines from the graph of a function: domain, range, zeros, intervals of monotonicity, intervals where the function takes values greater than (not less than) or less than (not greater than) a given number, maximum and minimum values (if they exist) in a closed interval, and arguments for which these values are attained;

5. interprets coefficients in the formula of a linear function;

6. determines the formula of a linear function based on information about its graph or properties;

7. sketches the graph of a quadratic function given by its formula;

8. interprets coefficients in the general, vertex, and factored forms of a quadratic function (if they exist);

9. determines the formula of a quadratic function based on information about the function or its graph;

10. finds the maximum and minimum value of a quadratic function in a closed interval;

11. applies the properties of linear and quadratic functions to interpret geometric, physical, and practical problems;

12. sketches graphs of functions $y = f(x - a)$, $y = f(x) + b$, $y = -f(x)$, $y = f(-x)$ based on the graph of $y = f(x)$;

13. uses the function $f(x) = \dfrac{a}{x}$ and its graph to describe and interpret problems involving inverse proportionality, including practical applications;

14. uses exponential and logarithmic functions and their graphs to describe and interpret problems related to practical applications.

VI. Sequences

The student:

1. calculates terms of a sequence defined by a general formula;

2. calculates initial terms of sequences defined recursively, such as:

(a)
$$\begin{cases} a_1 = 0.001 \\ a_{n+1} = a_n + \dfrac{1}{2}a_n(1 - a_n) \end{cases}$$

(b)
$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_{n+2} = a_{n+1} + a_n \end{cases}$$

3. determines whether a sequence is increasing or decreasing in simple cases;

4. identifies whether a given sequence is arithmetic or geometric;

5. applies the formula for the n-th term and the sum of the first n terms of an arithmetic sequence;

6. applies the formula for the n-th term and the sum of the first n terms of a geometric sequence;

7. applies properties of sequences, including arithmetic and geometric ones, to solve problems, including those with practical context.

VII. Trigonometry

The student:

1. uses definitions of the sine, cosine, and tangent functions for angles from $0°$ to $180°$, in particular calculates values for $30°$, $45°$, and $60°$;

2. finds approximate values of trigonometric functions using tables or a calculator;

3. finds the approximate value of an angle given the value of its trigonometric function using tables or a calculator;

4. applies the identities $\sin^2 \alpha + \cos^2 \alpha = 1$ and $\tan \alpha = \dfrac{\sin \alpha}{\cos \alpha}$;

5. applies the Law of Sines and the Law of Cosines and the formula for the area of a triangle: $P = \dfrac{1}{2} \cdot a \cdot b \cdot \sin \gamma$;

6. calculates angles and side lengths in triangles when sufficient information is given (solves triangles).

VIII. Planar Geometry (Stereometry)

The student:

1. determines radii and diameters of circles, lengths of chords and tangents, including with the use of the Pythagorean theorem;

2. identifies acute, right, and obtuse triangles based on side lengths (e.g. using the converse of the Pythagorean theorem and the cosine rule); applies the theorem that the side opposite a larger interior angle is longer;

3. identifies regular polygons and uses their basic properties;

4. uses angle and diagonal properties of rectangles, parallelograms, rhombi, and trapezoids;

5. uses properties of inscribed and central angles;

6. applies formulas for the area of a circular sector and the length of an arc;

7. applies theorems such as: Thales' theorem, its converse, the angle bisector theorem, and the theorem on the angle between a tangent and a chord;

8. applies similarity criteria for triangles;

9. applies the relationships between the perimeters and areas of similar figures;

10. identifies key points in triangles: incenter, circumcenter, orthocenter, centroid, and uses their properties;

11. uses trigonometric functions to determine side lengths in plane figures and to calculate areas;

12. constructs geometric proofs.

IX. Analytic Geometry on the Cartesian Plane

The student:

1. determines the mutual position of lines in the plane based on their equations, including finding the point of intersection of two lines, if it exists;

2. uses equations of lines in slope-intercept and general form, including determining the equation of a line with given properties (e.g., passing through two points, given slope, parallel or perpendicular to another line, tangent to a circle);

3. calculates the distance between two points in the coordinate system;

4. uses the equation of a circle $(x - a)^2 + (y - b)^2 = r^2$;

5. calculates the distance from a point to a line;

6. finds the points of intersection between a line and a circle or between a line and a parabola (as the graph of a quadratic function);

7. determines images of circles and polygons under axial symmetries with respect to coordinate axes and central symmetry with respect to the origin.

X. Solid Geometry

The student:

1. identifies the mutual position of lines in space, particularly skew and perpendicular lines;

2. uses the concept of the angle between a line and a plane and the dihedral angle between two half-planes;

3. identifies in prisms and pyramids the angles between segments (e.g., edges, edges and diagonals) and angles between faces, calculates the measures of these angles;

4. identifies in cylinders and cones the angles between segments and between segments and planes (e.g., the apex angle of a cone, angle between a generator and the base), calculates the measures of these angles;

5. determines the shape of a cross-section of a cuboid by a plane;

6. calculates volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres, including with the use of trigonometry and known theorems;

7. applies the relationship between volumes of similar solids.

XI. Probability and Statistics

The student:

1. calculates probabilities using the classical model;

2. understands and interprets data using percentiles;

3. calculates the arithmetic and weighted means, determines the median and mode;

4. calculates the standard deviation of a data set (also for grouped data), interprets this parameter in empirical contexts;

5. calculates expected value, e.g., to determine expected winnings in simple games of chance and lotteries.

XII. Optimization Problems

The student solves optimization problems in situations describable by a quadratic function.

Content based on the 2023 CKE core curriculum for general secondary schools. `https://cke.gov.pl/images/_EGZAMIN_MATURALNY_OD_2023/podstawa_programowa/matematyka.pdf`